Epidemics in the New Keynesian Model*†

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Abstract

This paper documents the behavior of key macro aggregates in the wake of the Covid epidemic. We show that a unique feature of the Covid recession is that the peak-to-trough decline is roughly the same for consumption, investment, and output. In contrast to the 2008 recession, there was only a short-lived rise in financial stress that quickly subsided. Finally, there was mild deflation between the peak and the trough of the Covid recession. We argue that a New Keynesian model that explicitly incorporates epidemic dynamics captures these qualitative features of the Covid recession. A key feature of the model is that Covid acts like a negative shock to the demand for consumption and the supply of labor.

JEL Classification: E1, I1, H0
Keywords: Epidemic, comovement, investment, recession.

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†Replication codes are available on the authors' websites.

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1 Introduction

This paper documents the behavior of key macro aggregates in the wake of the Covid epidemic, using data for six industrialized countries. The Covid epidemic is associated with a large recession in which consumption, investment, and output comove positively. In contrast to the 2008 recession, there was only a short-lived rise in financial stress that quickly subsided. In addition, there was mild deflation between the peak and the trough of the Covid recession. A unique feature of the Covid recession is that the peak-to-trough decline is roughly the same for consumption, investment, and output.

What class of models can account for these stylized facts? In Eichenbaum, Rebelo, and Trabandt (2021) we argue that the Covid epidemic acts like a simultaneous negative shock to the demand for consumption goods and the supply of labor. We extend that framework to incorporate investment as well as nominal price rigidities standard in New Keynesian (NK) models. We show that the model captures the main qualitative features of the Covid recession, generating sizable declines of similar magnitude in consumption, investment, and output, along with moderate deflation.

The intuition for our results is as follows. Suppose that people can become infected through consumption activities but not by working. Then, an epidemic leads to a large drop in consumption and a boom in investment. The latter boom reflects two forces: people want to consume more once the infection wanes and they want to smooth hours worked over time. By building up the capital stock, they can accomplish both objectives. Investment rises and consumption falls, so this version of the model cannot account for the severity and the comovement properties of the Covid recession.

Now suppose that people can become infected by working but not through consumption activities. Then, an epidemic leads to a small decline in consumption but a large fall in hours worked and output. There is also a large fall in investment because people smooth consumption in the face of a transitory fall in income. This version of the model does produce a large recession. However, it cannot account for consumption falling by roughly as much as investment during the Covid recession.

In the calibrated version of the model, consistent with the empirical evidence (see e.g., Barbieri et al. (2020) and Cai et al. (2020)), people can become infected through both consumption and work-related activities. We find that the shift in labor supply dominates the shift in consumption demand. So, an epidemic generates a steep recession along with sharp declines in both consumption and investment. The model is also consistent with the mild deflation observed in the data.

We find that sticky prices increase the depth of the recession relative to a model with flexible prices. But this effect is relatively small. The intuition for this result is as follows. It is well known that nominal price rigidities exacerbate the impact of negative demand shifts. But they alleviate the impact of negative supply shifts.[1] Since both shifts are operative during an epidemic, sticky prices do not, on net, have a strong effect on the response of output to an epidemic. Nevertheless, sticky prices are important for the overall performance of the model. With flexible prices, the model overstates the deflation associated with the Covid recession.

Starting with Barro and King (1984), there is an extensive literature that emphasizes the role of different shocks and frictions in generating comovement between consumption, investment, and output. An important strand of this literature emphasizes the role of financial frictions (see the reviews of the literature provided by Arellano et al. (2019) and Di Tella and Hall (2021)). Our empirical results indicate that, in this recession, whatever stresses there were in financial markets they were very short lived. No doubt, this fact reflects the policy interventions undertaken by the Federal Reserve and the U.S. Treasury. But regardless, one cannot attribute the large drop in consumption and the comovement between investment and consumption to a large rise in the importance of financial frictions. For this reason, our model abstracts from such frictions. This modeling decision does not reflect the view that financial frictions were unimportant in other episodes like the Great Recession.

To articulate in a transparent way the impact of the epidemic on aggregate demand and supply, we focus on the first wave of Covid infections. For the same reason, we abstract from government interventions such as containment measures and transfers to the households. We view our work as a first step towards integrating epidemic models and DSGE models. Our efforts complement research that mimics the impact of an epidemic in DSGE models through a sequence of demand and supply shocks (see e.g., Faria-e-Castro (2021) and Chen et al. (2021)).

Our analysis relates to the work of Guerrieri et al. (2021). In their model, an epidemic is equivalent to a supply shock. These authors show that, in multi-sector models with nominal rigidities, a supply shock can trigger a demand shortage that leads to an aggregate

[1] See Woodford (2003) and Gali (2015) for classic discussions of the effect of demand and supply shocks in New Keynesian models.

contraction. In contrast, in our model, an epidemic necessarily leads to both a contraction in supply and demand.

The remainder of this paper is organized as follows. In Section 2, we summarize some key facts about the Covid recession. Section 3 describes an NK model with endogenous epidemic dynamics. Section 4 uses this model to study the role of demand and supply shocks induced by the epidemic. Section 5 concludes.

2 The Covid recession

In this section, we summarize the behavior of key economic aggregates during the Covid recession and compare it to their behavior in other recessions. Our primary focus is the U.S., but we show that our key findings hold for other developed economies.

2.1 Data

In this subsection, we discuss the data used in our analysis. We use quarterly data for the U.S., Canada, France, Italy, Germany, and the UK.

2.1.1 U.S. data sources

For the U.S., we obtain seasonally-adjusted data from the Bureau of Economic Analysis on real GDP (A191RX), real consumption of nondurable goods (DNDGRA), real expenditures on services (DSERRA), real expenditures on durable goods (DDURRA), real fixed residential investment (A011RA), and real non-residential investment (A008RA).[2] We compute real expenditures on nondurables and services by taking a weighted average of real consumption of nondurable goods and real expenditures on services. The weights are given by the beginning-of-sample share of nominal expenditures on nondurables goods (DNDGRC) and services (DSERRC) in total expenditures on nondurable goods and services. We construct a series for real investment by taking a weighted average of expenditures on real durable goods, fixed residential investment, and non-residential investment at the beginning of the sample of interest. The data on the headline Consumer Price Index (CPI) and the core CPI (CPILFESL and CPIAUCSL, respectively) is from the Bureau of Economic Analysis, obtained from the Federal Reserve Bank of St. Louis database (FRED). We obtain data on the FRB Chicago National Financial Conditions Index along with its sub-indices from the Federal

[2] These series can be obtained from the following link:
https://apps.bea.gov/iTable/iTable.cfm?reqid=19&step=2#reqid=19&step=2&isuri=1&1921=s

Reserve Bank of Chicago.[3] We also obtain data on the Federal Reserve Bank of St. Louis Financial Stress Index.[4] We use the dates for the beginning of the recessions determined by the NBER except for the Covid recession which we time as starting in 2019.Q4.

2.1.2 Canadian data sources

For Canada, we obtain seasonally-adjusted data from Statistics Canada on real GDP (v62305752), real consumption of nondurable goods (v62305728), real consumption of semi-durable goods (v62305727), real expenditures on services (v62305729), and real expenditures on durable goods (v62305726).[5]

We compute real expenditures on nondurable goods and services using the beginning-of-sample weighted average of the share of nominal expenditures on nondurable goods (v62305759), semi-durable goods (v62305758), and services (v62305760) in total nominal expenditures on nondurable goods, semi-durable goods and services. We compute total real investment as the weighted sum of real expenditures on residential Investment (v62305734), real investments of non-profit institutions serving households (v62305739), machinery and equipment (v62305735), intellectual property products (v62305738), and durable goods (v62305726). The weights are given by the beginning-of-sample share of nominal expenditures on residential investment (v62305765), non-profit institutions (v62305770), machinery and equipment (v62305766) plus intellectual property products (v62305769), and durable goods(v62305757) in total expenditures on these categories. We obtain data on the headline CPI (v41690973) and core CPI (v41755376) from the Bank of Canada.[6] We were unable to obtain a financial stress index for Canada. We use the C. D. Howe institute dates for the beginning of recessions in Canada.

2.1.3 UK Data Sources

For the UK, we obtain seasonally-adjusted data from the Office of National Statistics on real GDP (YBIL), real consumption of nondurable goods (UTIL), plus semi durables (UTIT), services (UTIP), and durable goods (UTID).[7] We compute aggregate real nondurable and services using the beginning-of-sample weighted average of the share in total nominal expenditures on nondurable goods (UTIJ), semi-durables (UTIR), and services (UTIN).

[3] https://www.chicagofed.org/publications/nfci/index
[4] https://fred.stlouisfed.org/series/STLFSI2
[5] https://www150.statcan.gc.ca/n1/pub/13-607-x/2016001/64-eng.htm
[6] https://www.bankofcanada.ca/rates/price-indexes/cpi
[7] https://www.ons.gov.uk/

We compute real total investment plus durable goods as the weighted sum of total fixed capital formation (NPQT) and real durable goods (UTID). The weights are given by the share of nominal expenditures in total fixed capital formation (NPQS) and durable goods (UTIB) in total nominal expenditures on total fixed capital formation and durable goods. We obtain data on the headline CPI (CPIH) and core CPI (DKC6) from the Office of National Statistics. We use the Economic Cycle Research Institute for the beginning of recessions in the UK.

2.1.4 Data for France, Germany, and Italy

For France, Germany, and Italy we obtain data on nominal and real GDP as well as consumption, all investment categories, and headline and core CPI from Eurostat.[8]. We compute aggregate real categories as weighted averages of the underlying nominal categories using the same method as for the U.S., Canada, and the UK. Our financial stress indices for the UK, Germany, France, and Italy are from the European Central Bank.[9] We use the Economic Cycle Research Institute dates for the beginning of recessions.

2.2 Empirical results

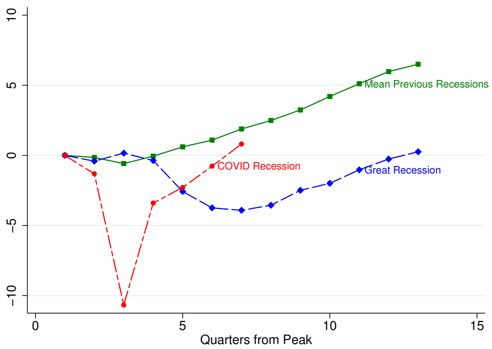
We begin by discussing our results for the U.S. Figures 1 and 2 display the behavior of key macro aggregates during the Covid recession, the Great Recession that started in the fall of 2008, and the average of the recessions that occurred between 1947 Q1 and 2008 Q3.

[8] We downloaded data on consumption, investment and price indeces from the following three links:
https://ec.europa.eu/eurostat/databrowser/view/namq_10_gdp/default/table?lang=en
https://ec.europa.eu/eurostat/databrowser/view/namq_10_an6/default/table?lang=en
https://ec.europa.eu/eurostat/databrowser/view/prc_hicp_midx/default/table?lang=en

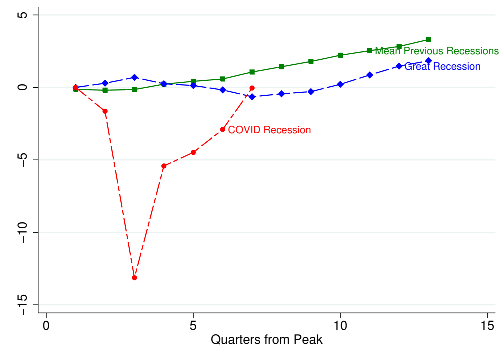
[9] https://sdw.ecb.europa.eu/browse.do?node=9693347

Figure 1: Main Macroeconomic Aggregates: U.S.

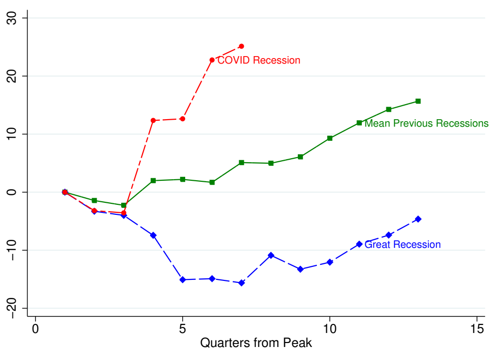
(a) *Real GDP*



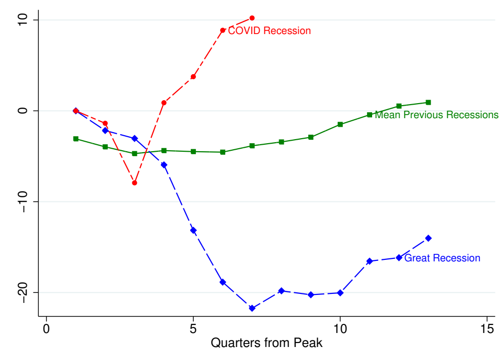
(b) *Real Expenditure on Non-Durables & Services*



(c) *Real Expenditure on Durables*



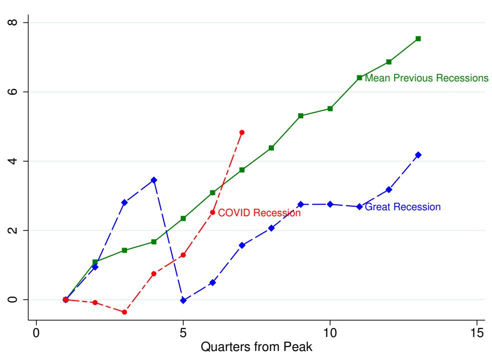
(d) *Real Expenditure on Durables & Investments*



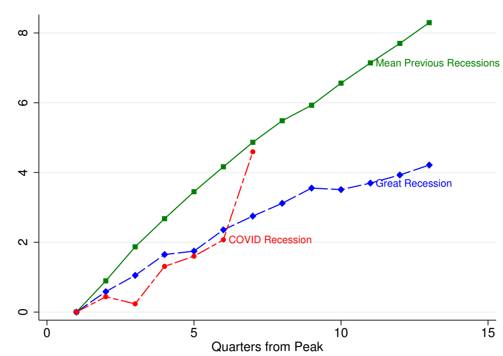
Note: y-axis expressed in percent deviation from recession peaks. See text for data sources.

Figure 2: Price Indexes: U.S.

(a) *Headline Consumer Price Index*



(b) *Core Consumer Price Index*



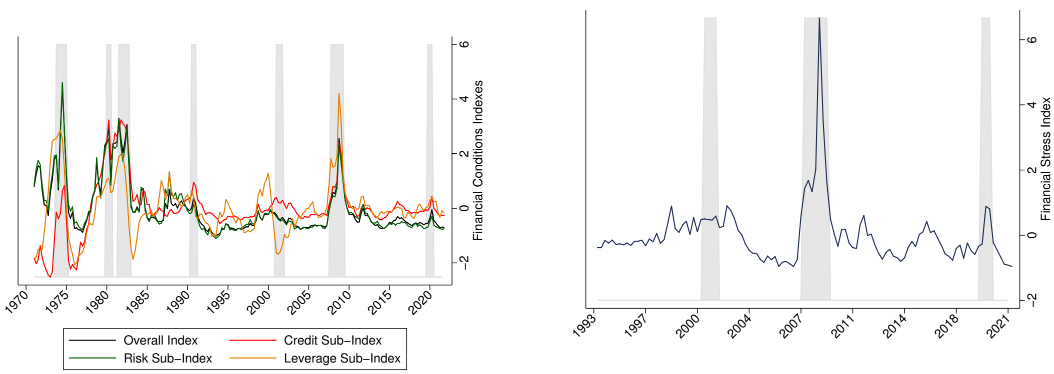
Note: y-axis expressed in percent deviation from recession peaks. See text for data sources.

The variables displayed are real GDP, real consumption of non-durables and services, real fixed investment, purchases of durables goods, the consumer price index, and the core consumer price index. In all cases, we graph percent deviations of the variables from their values at the beginning of the recession. Figure 3 displays the Federal Reserve Bank of St. Louis Financial Stress Index and the Federal Reserve Bank of Chicago National Financial Conditions Index along with its subcomponents.

Figure 3: US Indexes of Financial Conditions and Financial Stress

(a) *Chicago FED Index of National Financial Conditions*

(b) *St. Louis FED Index of Financial Stress*



Note: Shaded areas indicate NBER recessions (peak to trough). See text for data sources.

Table 1 reports the percentage decline from peak to trough for real GDP, real consumption of non-durables and services, and real fixed investment plus purchases of durables goods for the Covid recession, the Great Recession, and the average of the other post-war recessions. For the real variables, troughs are specific to the variables in question. For the nominal variables (the headline and core CPI) we compute the percentage change from peak to trough in real GDP.

Table 1: *US: Peak to Trough (Variable Specific) Percentage Point Decrease for Main Aggregates.*

Recessions	GDP	Durables and Fixed Inv	Non-Durables and Services
Mean Pre-GFC	-0.69	-4.71	-0.19
GFC	-3.92	-21.73	-0.64
COVID-19	-10.67	-7.93	-13.14

Note: All estimates are percentage point deviations from level of the variable at the peak. The first two composite measures are weighted by the nominal values of the respective variables observed at the peak. GFC refers to Global Financial Crisis.

Several key results emerge for the U.S. First, the Covid recession was much deeper than other recessions. The peak-to-trough decline in real GDP is 15 times larger than in the average pre-2008 recession and about 2.7 times larger than in the Great Recession. Second, as is well known, in pre-Covid recessions consumption fell by less than output and by much less than investment. For example, in the Great Recession, the fall in consumption is about 6 times *smaller* than the fall in real GDP and about 34 times *smaller* than the fall in investment. In sharp contrast, in the Covid recession, the fall in consumption was 23 percent *larger* than the fall in output and 66 percent *larger* than the drop in investment. Third, nominal prices remained relatively stable during the Covid recession and recovered quickly thereafter (see Figure 2). Headline and core inflation from peak to trough are -0.8 percent and 0 percent, respectively. Fourth, according to the stress indices depicted in Figure 3, there was a rise in financial stress that quickly subsided.

Tables 2–6 report results for the other countries that we consider. While magnitudes vary across countries, the same key features emerge. The Covid recession was very large with unusually large drops in consumption relative to real GDP and investment. Movements in nominal prices during the recession were relatively small.

Table 2: *Canada: Peak to Trough (Variable Specific) Percentage Point Decrease for Main Aggregates.*

Recessions	GDP	Durables and Fixed Inv	Non-Durables and Services
Mean Pre-GFC	-2.4	-12.2	-0.48
GFC	-4.55	-16.1	-0.59
COVID-19	-14.03	-19.34	-16.49

Note: All estimates are percentage point deviations from level of the variable at the peak. The first two composite measures are weighted by the nominal values of the respective variables observed at the peak. GFC refers to Global Financial Crisis.

Table 3: *United Kingdom: Peak to Trough (Variable Specific) Percentage Point Decrease for Main Aggregates.*

Recessions	GDP	Durables and Fixed Inv	Non-Durables and Services
Mean Pre-GFC	-4.38	-14.58	-0.35
GFC	-5.66	-15.84	-4.22
COVID-19	-24.51	-25.2	-25.18

Note: All estimates are percentage point deviations from level of the variable at the peak. The first two composite measures are weighted by the nominal values of the respective variables observed at the peak. GFC refers to Global Financial Crisis.

Table 4: *Germany: Peak to Trough (Variable Specific) Percentage Point Decrease for Main Aggregates.*

Recessions	GDP	Durables and Fixed Inv	Non-Durables and Services
Mean Pre-GFC	-1.38	-10.32	-0.66
GFC	-7.05	-8.13	-0.7
COVID-19	-12.31	-8.82	-11.03

Note: All estimates are percentage point deviations from level of the variable at the peak. The first two composite measures are weighted by the nominal values of the respective variables observed at the peak. GFC refers to Global Financial Crisis.

Table 5: *France: Peak to Trough (Variable Specific) Percentage Point Decrease for Main Aggregates.*

Recessions	GDP	Durables and Fixed Inv	Non-Durables and Services
Mean Pre-GFC	-0.29	-2.25	-0.07
GFC	-3.93	-2.69	-0.59
SOV	0	-1.64	0
COVID-19	-20.31	-13.48	-18.96

Note: All estimates are percentage point deviations from level of the variable at the peak. The first two composite measures are weighted by the nominal values of the respective variables observed at the peak. GFC refers to Global Financial Crisis.

Table 6: *Italy: Peak to Trough (Variable Specific) Percentage Point Decrease for Main Aggregates.*

Recessions	GDP	Durables and Fixed Inv	Non-Durables and Services
Mean Pre-GFC	-0.81	-0.95	-0.76
GFC	-7.89	-15.73	-2.96
SOV	-5.53	-23.59	-6.21
COVID-19	-19.66	-33.46	-21.54

Note: All estimates are percentage point deviations from level of the variable at the Peak. The first two composite measures are weighted by the nominal values of the respective variables observed at the Peak. GFC refers to Global Financial Crisis.

Table 7 reports the average peak-to-trough change in real GDP, investment, consumption, and the consumer price index across our six economies in the Covid recession. We see that the average decline in all real variables is about the same. This fact contrasts sharply with the relative behavior of these variables in other recessions.

Table 7: *Peak to Trough (Variable Specific) Percentage Point Decrease for Main Consumption and Investment Aggregates.*

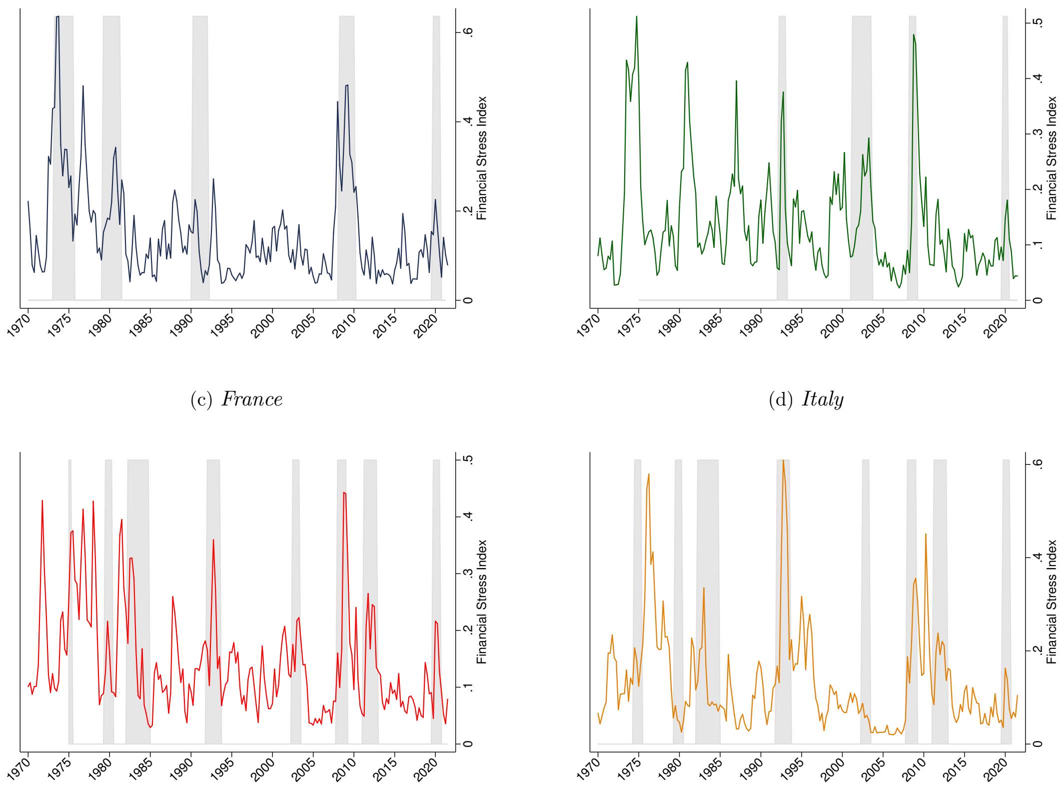
Countries	GDP	Durables and Fixed Inv	Non-Durables and Services
US	-10.67	-7.93	-13.14
CA	-14.03	-19.34	-16.49
UK	-24.51	-25.2	-25.18
DE	-12.31	-8.82	-11.03
FR	-20.31	-13.48	-18.96
IT	-19.66	-33.46	-21.54
Average	-16.92	-18.04	-17.72

Note: All estimates are percentage point deviations from level of the variable at the peak. The first two composite measures are weighted by the nominal values of the respective variables observed at the peak. GFC refers to Global Financial Crisis.

Finally, Figure 4 depicts financial stress indices for France, Germany, Italy, and the UK.[10] As in the U.S., there was a modest rise in the level of financial stress that quickly subsided probably as a result of the response of governments and central banks to the crisis.

[10] We could not find a comparable index for Canada.

Figure 4: Country Level Indexes of Financial Stress



(a) *United Kindom* (b) *Germany* (c) *France* (d) *Italy*

Note: Shaded areas indicate ECRI recessions (peak to trough). See text for data sources.

3 The model economy

In this section, we show that a simple NK model, augmented to explicitly incorporate the non-linear dynamics of an epidemic, can account for the key facts documented in Section 2. The ability of the model to do so reflects the view that an epidemic is associated with negative shocks to both consumption demand and labor supply.

To make our argument as transparent as possible, we focus on a simple model of the epidemic in which there is a single wave of infections. For the reasons discussed in the previous section, we purposefully abstract from financial frictions. These frictions almost certainly played a role in generating comovement between investment and consumption in the Great Recession. But they are an unlikely candidate for an explanation of comovement during the Covid recession.

We also abstract from containment measures introduced in various countries. We do so for two reasons. First, during the epidemic, there was positive comovement between consumption, investment, and output both in periods with and without containment. Second, modeling those measures would considerably complicate the analysis without changing our basic message. The reason is that containment acts as an additional negative shock to the demand for consumption and supply of labor (see e.g. Eichenbaum, Rebelo, and Trabandt (2021)).

The economy is initially in a steady state where all people are identical. The population is then divided into four groups: susceptible (people who have not yet been exposed to the virus), infected (people who have been infected by the virus), recovered (people who survived the infection and acquired immunity), and deceased (people who died from the infection). We denote the fraction of the initial population in each group by S_t, I_t, R_t, and D_t, respectively. The variable T_t denotes the number of newly infected people.

At time zero, a fraction ε of the population is infected by a virus:

$$I_0 = \varepsilon.$$

The rest of the population is susceptible to the virus:

$$S_0 = 1 - \varepsilon.$$

Social interactions occur at the beginning of the period (infected and susceptible people meet). Then, changes in health status unrelated to social interactions (recovery or death) occur. At the end of the period, the consequences of social interactions materialize and T_t susceptible people become infected.

As in Eichenbaum, Rebelo and Trabandt (2021), we assume that susceptible people can become infected in three ways: purchasing consumer goods, working, and through random interactions unrelated to economic activity. The number of newly infected people is given by the transmission function:[11]

$$T_t = \pi_1 \left(S_t C_t^s\right) \left(I_t C_t^i\right) + \pi_2 \left(S_t N_t^s\right) \left(I_t N_t^i\right) + \pi_3 S_t I_t. \tag{1}$$

The variables C_t^s and C_t^i represent the consumption of a susceptible and infected person, respectively. The variables N_t^s and N_t^i represent hours worked of a susceptible and infected

[11] To simplify, we assume that the probability of a given person being infected through more than one form of social interactions is zero. In addition, we do not explicitly incorporate the constraint that T_t must be between zero and the size of population. This constraint is satisfied in all our simulations.

person, respectively. The number of newly infected people that results from consumption-related interactions is given by $\pi_1 \left(S_t C_t^s\right) \left(I_t C_t^i\right)$. The terms $S_t C_t^s$ and $I_t C_t^i$ represent total consumption of susceptible and infected people, respectively. The parameter π_1 reflects both the amount of time spent in consumption activities and the probability of becoming infected as a result of those activities.

The number of newly infected people that results from interactions at work is given by $\pi_2(S_t N_t^s) \left(I_t N_t^i\right)$. The terms $S_t N_t^s$ and $I_t N_t^i$ represent total hours worked by susceptible and infected people, respectively. The parameter π_2 reflects the probability of becoming infected as a result of work interactions.

Susceptible and infected people can meet in ways unrelated to consuming or working. The number of random meetings between infected and susceptible people is $S_t I_t$. These meetings result in $\pi_3 S_t I_t$ newly infected people. The number of susceptible people at time $t+1$ is given by:

$$S_{t+1} = S_t - T_t. \tag{2}$$

The number of infected people at time $t+1$ is equal to the number of infected people at time t plus the number of newly infected people (T_t) minus the number of infected people who recovered $(\pi_r I_t)$ and the number of infected people who died $(\pi_d I_t)$:

$$I_{t+1} = I_t + T_t - \left(\pi_r + \pi_d\right) I_t. \tag{3}$$

Here, π_r is the rate at which infected people recover from the infection and π_d is the probability that an infected person dies.

The number of recovered people at time $t+1$ is the number of recovered people at time t plus the number of infected people who just recovered $(\pi_r I_t)$:

$$R_{t+1} = R_t + \pi_r I_t. \tag{4}$$

Finally, the number of deceased people at time $t+1$ is the number of deceased people at time t plus the number of new deaths $(\pi_d I_t)$:

$$D_{t+1} = D_t + \pi_d I_t. \tag{5}$$

People have rational expectations, so that they are aware of the initial infection and understand the laws of motion governing population health dynamics.

Final good producers Final output, Y_t, is produced by a representative, competitive firm using the technology:

$$Y_t = \left(\int_0^1 Y_{i,t}^{\frac{1}{\gamma}} di \right)^{\gamma}, \gamma > 1. \tag{6}$$

The variable $Y_{i,t}$ denotes the quantity of intermediate input i used by the firm.

Profit maximization implies the following demand schedule for intermediate products:

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\frac{\gamma}{\gamma-1}} Y_t. \tag{7}$$

Here, $P_{i,t}$ denotes the price of intermediate input i in units of the final good.

The price of output is given by:

$$P_t = \left(\int_0^1 P_{i,t}^{-\frac{1}{\gamma-1}} di \right)^{-(\gamma-1)}.$$

Intermediate goods producers Intermediate good i is produced by a monopolist using labor, $N_{i,t}$, and capital, $K_{i,t}$, according to the technology:

$$Y_{i,t} = AK_{i,t}^{1-\alpha} N_{i,t}^{\alpha}.$$

The profits of intermediate-good producer i at time t are:

$$\pi_{i,t} = P_{i,t} Y_{i,t} - P_t mc_t Y_{i,t}.$$

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Monopolist i chooses its price subject to Calvo (1983) style price-setting frictions. With probability $1-\xi$ the firm reoptimizes $P_{i,t}$. With probability ξ, $P_{i,t} = P_{i,t-1}$. The firm chooses its optimal time-t price, $\tilde{P}_t$, to maximize:

$$\max_{\tilde{P}_t} \sum_{j=0}^{\infty} (\xi\beta)^j \lambda_{t+j}^b \left(\tilde{P}_t Y_{i,t+j} - P_{t+j} mc_{t+j} Y_{i,t+j} \right),$$

subject to the demand curve (7). Here, λ_{t+j}^b is the Lagrange multiplier of the household problem associated with the nominal budget constraint and mc_t denotes the real marginal cost at time t:

$$mc_t = \frac{W_t^{\alpha} \left(R_t^k \right)^{1-\alpha}}{P_t A \alpha^{\alpha} (1-\alpha)^{1-\alpha}},$$

where W_t and R_t^k are the nominal wage rate and rental rate of capital, respectively.

Households For tractability, we assume that people are organized into households, each of which has a continuum of identical members. This household structure introduces limited sharing of health risks. Without the household structure, the asset holdings of a person would depend on how long they had a particular health status. So, as time goes by, we would have to keep track of an increasing number of types of people.

At time zero, a household has a continuum of measure one of family members. The household maximizes its lifetime utility:

$$U = \sum_{t=0}^{\infty} \beta^t \left\{ s_t \left[\log(c_t^s) - \frac{\theta}{2} \left(n_t^s\right)^2 \right] + i_t \left[\log(c_t^i) - \frac{\theta}{2} \left(n_t^i\right)^2 \right] + r_t \left[\log(c_t^r) - \frac{\theta}{2} \left(n_t^r\right)^2 \right] \right\}, \quad (8)$$

subject to the budget constraint:

$$B_{t+1} + P_t \left(s_t c_t^s + i_t c_t^i + r_t c_t^r + x_t \right) + \Psi_t = R_{t-1}^b B_t + W_t \left(s_t n_t^s + i_t n_t^i + r_t n_t^r \right) + R_t^k k_t + \Phi_t. \quad (9)$$

Here, s_t, i_t, and r_t denote the measure of family members who are susceptible, infected and recovered. The variables $(c_t^s,\, c_t^i,\, c_t^r)$ and $(n_t^s,\, n_t^i,\, n_t^r)$ denote the consumption and hours worked of susceptible, infected and recovered family members, respectively. The variables Φ_t and Ψ_t denote profits from the monopolistically competitive firms and lump-sum taxes, respectively. The variable x_t denotes household capital investment. The variable B_t denotes nominal bond holdings, and R_t^b the interest rate on nominal bonds.

The law of motion for the stock of capital is:

$$k_{t+1} = x_t + (1 - \delta) k_t. \quad (10)$$

The number of newly infected people is given by:

$$\tau_t = \pi_1 s_t c_t^s \left(I_t C_t^i \right) + \pi_2 s_t n_t^s \left(I_t N_t^i \right) + \pi_3 s_t I_t. \quad (11)$$

The household can affect τ_t through its choice of c_t^s and n_t^s. However, the household takes economy-wide aggregates $I_t C_t^I$, and $I_t N_t^I$ as given, i.e. it does not internalize the impact of its choices on economy-wide infection rates.

The fraction of the initial family that is susceptible, infected and recovered at time $t+1$ is given by:

$$\begin{aligned} s_{t+1} &= s_t - \tau_t, & (12) \\ i_{t+1} &= i_t + \tau_t - (\pi_r + \pi_d)\, i_t, & (13) \\ r_{t+1} &= r_t + \pi_r i_t. & (14) \end{aligned}$$

The first-order conditions for c_t^s, c_t^i and c_t^r are:

$$\begin{aligned} \frac{1}{c_t^s} &= \lambda_t^b P_t - \lambda_t^\tau \pi_1 \left(I_t C_t^I\right), \\ \frac{1}{c_t^i} &= \lambda_t^b P_t, \\ \frac{1}{c_t^r} &= \lambda_t^b P_t. \end{aligned}$$

Here, λ_t^b and λ_t^τ are the Lagrange multiplier on the household budget constraint and the transmission function (11), respectively. The first-order conditions for n_t^s, n_t^i and n_t^r are:

$$\begin{aligned} \theta n_t^s &= \lambda_t^b W_t + \lambda_t^\tau \pi_2 \left(I_t N_t^I\right), \\ \theta n_t^i &= \lambda_t^b W_t, \\ \theta n_t^r &= \lambda_t^b W_t. \end{aligned}$$

The first-order condition for k_{t+1} is:

$$\lambda_t^b P_t = \beta \lambda_{t+1}^b [R_{t+1}^k + P_{t+1}(1-\delta)]. \tag{15}$$

The first-order conditions for s_{t+1}, i_{t+1}, r_{t+1}, and τ_t are:

$$\begin{aligned} &\log(c_{t+1}^s) - \frac{\theta}{2}\left(n_{t+1}^s\right)^2 + \lambda_{t+1}^\tau \left[\pi_1 c_{t+1}^s \left(I_{t+1} C_{t+1}^I\right) + \pi_2 n_{t+1}^s \left(I_{t+1} N_{t+1}^I\right) + \pi_3 I_{t+1}\right] \\ &+\lambda_{t+1}^b \left[W_{t+1} n_{t+1}^s - P_{t+1} c_{t+1}^s\right] - \lambda_t^s/\beta + \lambda_{t+1}^s = 0, \end{aligned}$$

$$\begin{aligned} &\log(c_{t+1}^i) - \frac{\theta}{2}\left(n_{t+1}^i\right)^2 + \\ &+\lambda_{t+1}^b \left[W_{t+1} n_{t+1}^i - P_{t+1} c_{t+1}^i\right] - \lambda_t^i/\beta + \lambda_{t+1}^i \left(1 - \pi_r - \pi_d\right) \\ &+\lambda_{t+1}^r \pi_r = 0, \end{aligned}$$

$$\begin{aligned} &\log(c_{t+1}^r) - \frac{\theta}{2}\left(n_{t+1}^r\right)^2 + \\ &+\lambda_{t+1}^b (W_{t+1} n_{t+1}^r - P_{t+1} c_{t+1}^r) - \lambda_t^r/\beta + \lambda_{t+1}^r = 0, \end{aligned}$$

$$-\lambda_t^\tau - \lambda_t^s + \lambda_t^i = 0.$$

Monetary policy The monetary authority controls the nominal interest rate, R_t^b. It chooses this rate according to the following Taylor-type rule:

$$R_t^b - R^b = \theta_\pi \log \frac{\pi_t}{\pi} + \theta_x \log(Y_t / Y_t^f),$$

where Y_t^f, is output in a flexible-price version of the economy. The variables π, and R_t^b are the steady-state values of the rate of inflation and the nominal interest rate, respectively.

Fiscal policy Real government spending, G, is constant over time and is financed with lump-sum taxes, Ψ_t.

Equilibrium In equilibrium, the market for goods and hours worked clear, households and firms solve their maximization problems.

The fraction of people in the family who are susceptible, infected and recovered is the same as the corresponding fraction in the population:

$$s_t = S_t,\ i_t = I_t, \text{ and } r_t = R_t.$$

Labor demand is equal to labor supply:

$$s_t n_t^s + i_t n_t^i + r_t n_t^r = N_t.$$

The demand for final goods equals the final goods supply:

$$AK_t^{1-\alpha} N_t^{\alpha} = C_t + X_t + G,$$

where K_t is the aggregate supply of capital and C_t, X_t and G are aggregate consumption, investment, and government expenditures, respectively. Consumption and investment are given by:

$$C_t = s_t c_t^s + i_t c_t^i + r_t c_t^r,$$

$$X_t = x_t.$$

The law of motion for the aggregate capital stock is:

$$K_{t+1} = X_t + (1-\delta) K_t.$$

In equilibrium, the market for physical capital clears

$$K_t = k_t,$$

Since nominal bonds are in zero net supply, in equilibrium:

$$B_t = 0.$$

The appendix contains the list of model equilibrium conditions.

3.1 Parameter values

Each time period corresponds to a week. We assume that it takes on average 14 days to either recover or die from the infection. Since our model is weekly, we set $\pi_r + \pi_d = 7/14$. Based on data for South Korea for people younger than 65 years, we choose the mortality rate to be 0.2 percent which implies $\pi_d = 7 \times 0.002/14$. With this value for the mortality rate, the model can account for the peak-to-trough decline in GDP during the Covid recession in the U.S.

We set π_1, π_2, and π_3 to 2.5684×10^{-7}, 1.5936×10^{-4}, and 0.4997, respectively. These values imply that in the beginning of the epidemic 1/6 of the virus transmissions come from consumption, 1/6 come from work and 2/3 come from non-economic activities:

$$\frac{\pi_1 C^2}{\pi_1 C^2 + \pi_2 N^2 + \pi_3} = 1/6, \tag{16}$$

$$\frac{\pi_2 N^2}{\pi_1 C^2 + \pi_2 N^2 + \pi_3} = 1/6. \tag{17}$$

Here, C and N denote consumption and hours worked in the pre-epidemic steady state, respectively. We choose the level of π_1 so that the Kermack and McKendrick (1927) SIR model is consistent with the "Merkel scenario" outlined by Chancellor Angela Merkel in her speech on March 11, 2020 (Bennhold and Eddy 2020). According to this scenario, 60 percent of the population ends up infected by the virus in the absence of any actions, public or private, to contain the virus.

Tables 8 and 9 display our parameter values as well as the values of key aggregate steady-state variables, respectively. The initial population is normalized to one. The number of people who are initially infected, ε, is 0.001. We choose $A = 2.148$ and $\theta = 0.001$ so that in the pre-epidemic steady state the representative person works 28 hours per week and earns a weekly income of $\$58,000/52$. We set the weekly discount factor, β, to $0.98^{1/52}$ so that the value of a life is about 11 million 2019 dollars in the pre-epidemic steady state. This value is consistent with the economic value of life used by U.S. government agencies (see Viscusi and Aldy (2003) for a discussion). We set the weekly depreciation rate, $\delta = 0.06/52$ and the labor share, $\alpha = 2/3$. We set the parameter that determines the markup, γ, to 1.35. This value is consistent with the range of estimates reported in Christiano, Eichenbaum and Trabandt (2016). The steady-state real wage is 19.67.

The steady-state share of government spending to GDP is set to 19 percent, a value that corresponds to the average share of government expenditures in the U.S. economy. These parameter values imply that the share of investment as a fraction of GDP is 18.4 percent. This share corresponds roughly to the average share of investment in GDP in the U.S. economy.

We assume that $\xi = 0.98$ so that prices change on average once a year. The coefficients in the Taylor rule are $\theta_\pi = 1.5$ and $\theta_x = 0.5/52$.

Table 8: Parameters and Steady-State Calibration Targets

Parameter	Value	Description
π_d	0.001	Probability of dying (weekly)
π_r	0.499	Probability of recovering (weekly)
ε	0.001	Initial infection
δ	0.06/52	Capital depreciation rate (weekly)
α	2/3	Marginal product of labor
γ	1.35	Gross price markup
ξ	0.98	Calvo price stickiness (weekly)
r_π	1.5	Taylor rule coefficient inflation
r_x	0.5/52	Taylor rule coefficient output gap
η	0.19	Gov. consumption share of output
n	28	Hours worked (weekly)
y	58000/52	Income (weekly)

Table 9: Steady States and Model Parameters

π_1	2.568×10^{-7}
π_2	1.593×10^{-4}
π_3	0.4997
A	2.148
θ	0.001
c/y	0.625
x/y	0.184
g/y	0.190
VoL	1.1×10^7
k/y	3.07
y	1115.3
c	697.4
x	206.0
g	211.9
w	19.6
k	178551
r^k	0.00154
λ^b	0.00143
λ^τ	−30.56
R^b	1.00039
π	1
mc	0.74

Notes: VoL denotes value of life. k/y is expressed in annual terms.

4 The impact of an epidemic

In this section, we discuss the impact of an epidemic in the model. Our parameterization of the transmission function (1) implies that an epidemic can be thought of as giving rise to negative aggregate demand and aggregate supply shocks. The aggregate demand shock arises because susceptible people reduce their consumption to lower their probability of being infected. A simple way to see this effect is to consider the first-order condition for c_t^s:

$$\frac{1}{c_t^s} = \lambda_t^b P_t - \lambda_t^\tau \pi_1 \left(I_t C_t^I \right). \tag{18}$$

Recall that $\lambda_t^b > 0$ is the Lagrange multiplier on the household budget constraint and $\lambda_t^\tau < 0$ is the Lagrange multiplier on τ_t. Other things equal, the larger is $\pi_1 \left(I_t C_t^I \right)$ the lower is c_t^s.

The negative aggregate supply shock arises because susceptible people reduce their hours worked to lower their probability of becoming infected. To see this effect, recall the first-order condition for n_t^s:

$$\theta n_t^s = \lambda_t^b W_t + \lambda_t^\tau \pi_2 \left(I_t N_t^I \right). \tag{19}$$

Other things equal, the larger is $\pi_2 \left(I_t N_t^I \right)$ the smaller is n_t^s.

Working in tandem, aggregate demand and supply shocks generate a prolonged recession. However, the qualitative and quantitative responses of consumption, hours worked and investment depend very much on which shock dominates.

The previous intuition about demand and supply shocks is suggestive about the first-order effects of the epidemic. There are other general equilibrium effects that must be considered. As it turns out, those effects do not overturn the intuition based on demand and supply shocks.

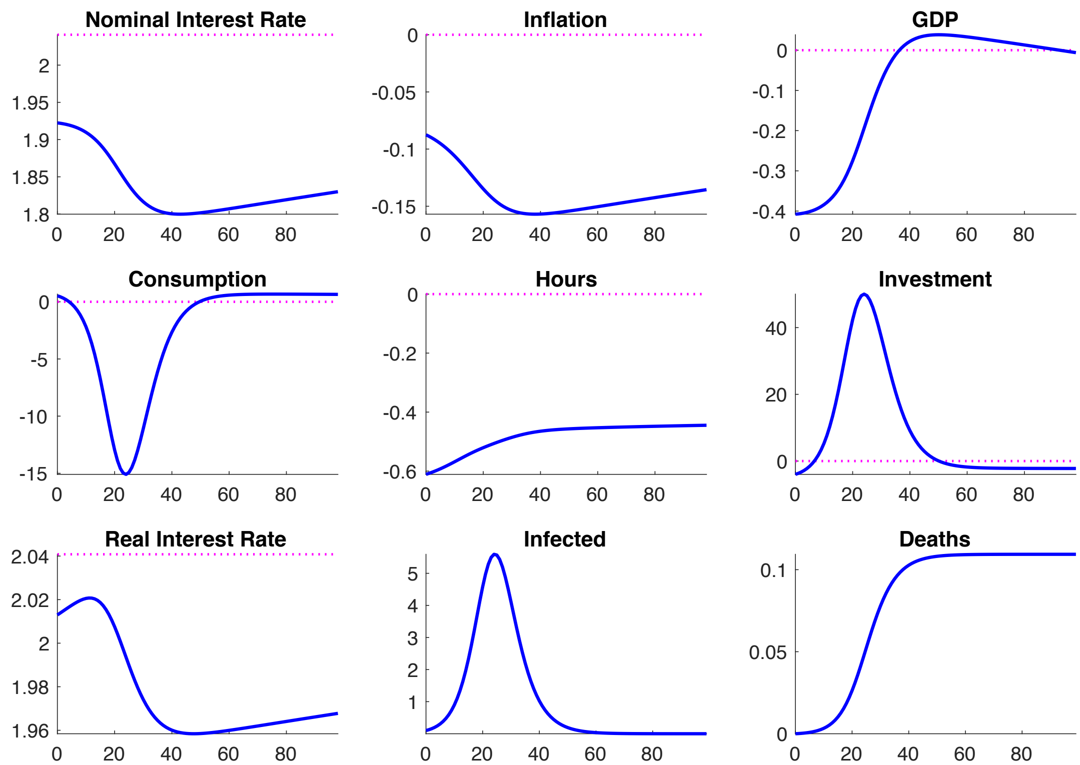
Subsections 4.1 and 4.2 focus on the effect of the shock to consumption demand and labor supply, respectively. In subsection 4.3, we combine the two shocks to assess the full impact of the epidemic.

4.1 Epidemics as a shock to the demand for consumption

To isolate the effect of the epidemic on consumption demand, we set π_2 to zero so that hours worked do not affect the probability of a susceptible person becoming infected. We calibrate π_1 to 6.3897×10^{-7} so that $1/3$ of the infections at the beginning of the epidemic are driven by consumption (see equation (16)).

Figure 5 displays the impact of the epidemic on key macro variables. The main results can be summarized as follows. First, there is a relatively small recession, with output and hours worked falling from peak to trough by 0.4 and 0.6 percent, respectively. Second, there is a very large drop in consumption (15 percent from peak to trough) and an enormous rise in investment (50 percent from trough to peak).

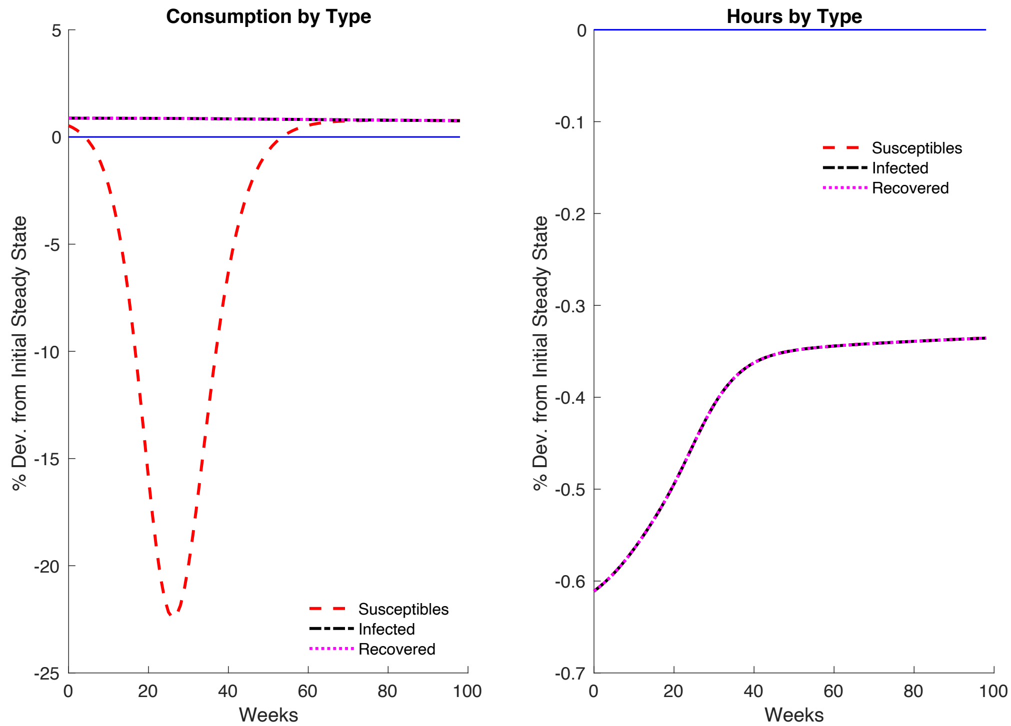
Figure 5: Epidemic as a shock to consumption demand



Notes: x-axis in weeks. GDP, consumption, hours and investment in percent deviations from initial steady state. Inflation, nominal and real interest rates in percent. Infected and deaths in percent of initial population.

Figure 6 shows consumption and hours worked for susceptible, infected, and recovered people. There is a large drop in the consumption of susceptible people (22 percent from peak to trough). This drop reflects people's desire to reduce the probability of becoming infected in the course of consuming market goods. In contrast, consumption of infected and recovered people rises by a small amount. Hours worked by susceptible, infected, and recovered people are relatively stable, exhibiting some dynamics that we discuss below.

Figure 6: Response of consumption and hours when epidemic is a shock to demand



The intuition for the results in Figures 5 and 6 is that the infection acts as a negative shock to susceptible people's demand for consumption. The household reduces c_t^s to lower the probability of susceptible people becoming infected. Consistent with this intuition, the path for c_t^s is the mirror image of the path for I_t.

The health status of infected and recovered people is not affected by exposure to the virus. So, their consumption demand does not shift down in response to movements in I_t. As a result, the household does not reduce c_t^i and c_t^r. In fact, they rise by a modest amount. To understand this response, note that the income of the household does not fall by very much. But c_t^s falls by a very large amount. The household uses a small part of the savings from the earnings of susceptible people to fund a small rise in c_t^i and c_t^r.

Figure 5 shows that the household uses most of those savings to finance a massive increase in investment. By building up the capital stock, the household makes it possible for c_t^s to rise once infections start to decline without large increases in n_t^s, n_t^i or n_t^r. In effect, investment allows the household to smooth the response of consumption and hours worked to a transitory shock in susceptible people's consumption demand.

Since most members of the household want to lower their consumption, the overall return to work declines. So, there is a small initial fall in hours worked. After a delay, hours worked rise, reflecting the increase in the marginal product of labor associated with the build up of capital.

In sum, when $\pi_2 = 0$, the epidemic generates a mild recession. But, with this parameterization, the model cannot rationalize two key features of the COVID-19 recession: the large drop in output and the positive comovement between investment and consumption.[12] It is possible that in a model with sticky wages in which employment is demand determined one could rationalize the features of the covid recession with $\pi_2 = 0$. However, there is strong evidence that infections happen through interactions in the workplace, i.e π_2 is positive (see e.g. Barbieri et al. (2020) and OSHA).[13]

4.2 Epidemics as a shock to the supply of labor

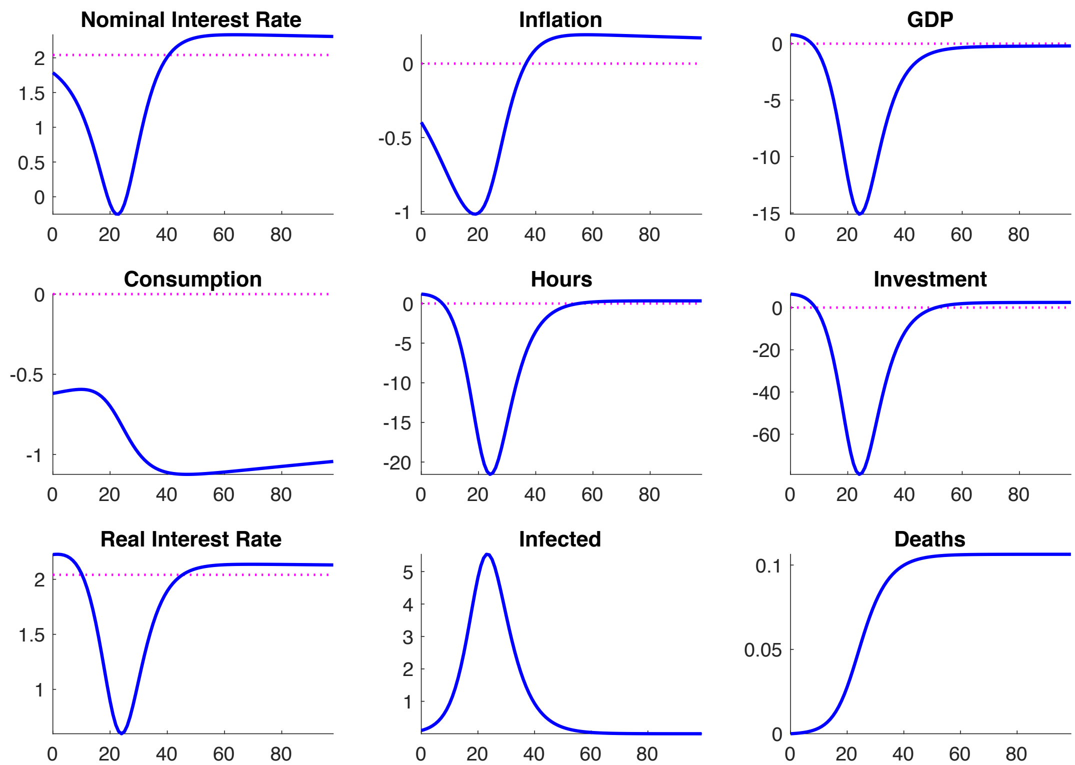
To isolate the effect of the epidemic on the supply of labor, we set π_1 to zero. With this parameterization, consumption does not affect the probability of a susceptible person becoming infected. We calibrate π_2 to 3.1871×10^{-4} so that 1/3 of the infections at the beginning of the epidemic (equation (17)) are driven by hours worked.

Figure 7 displays the impact of an epidemic on key macro variables. The epidemic causes a very large recession, with output and hours worked falling from peak to trough by 15 and 22 percent, respectively. Consumption declines modestly (1 percent from peak to trough), and there is a large fall in investment (79 percent from trough to peak).

[12] These declines in measures of economic activity occurred before lockdowns were imposed, as well as in countries like Sweden and South Korea, and U.S. states that did not impose lockdowns (see Andersen et al. (2020), Aum et al. (2020.) and Gupta et al. (2020)).

[13] https://www.osha.gov/coronavirus/hazards

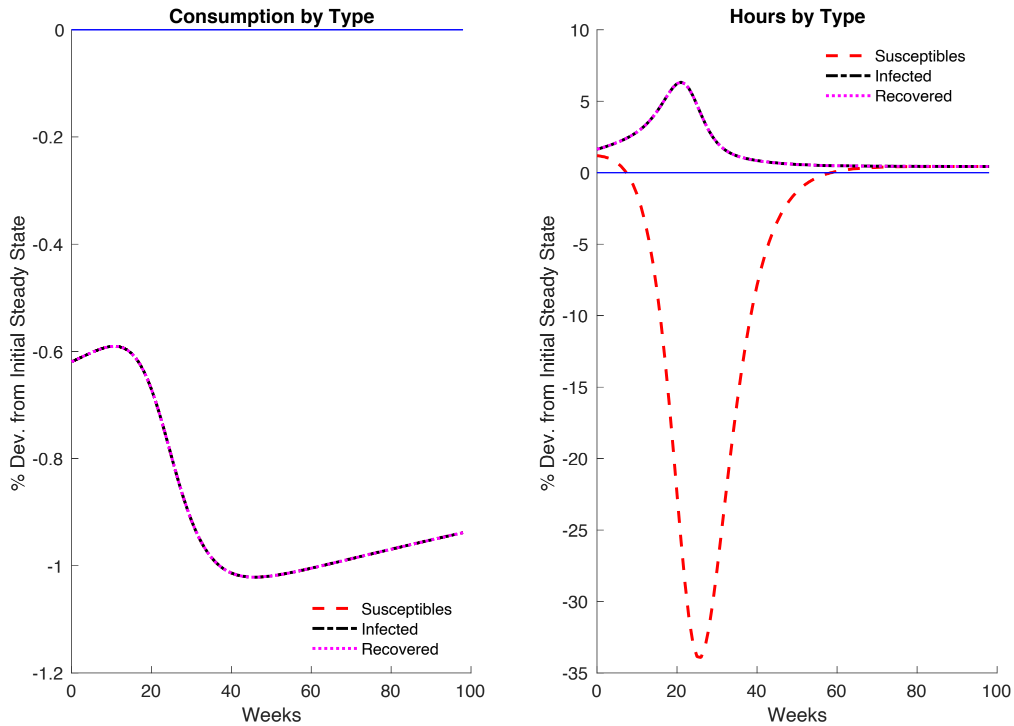
Figure 7: Epidemic as a shock to labor supply



Notes: x-axis in weeks. GDP, consumption, hours and investment in percent deviations from initial steady state. Inflation, nominal and real interest rates in percent. Infected and deaths in percent of initial population.

Figure 8 shows that c_t^s, c_t^i, and c_t^r all decline by the same small amount. In contrast, hours worked by different types of people respond very differently: n_t^s falls by 34 percent from peak to trough, while both n_t^i and n_t^r rise by 6 percent from trough to peak.

Figure 8: Response of consumption and hours when epidemic is a shock to supply



As discussed above, when $\pi_1 = 0$, the infection acts as a negative shock to susceptible people's supply of labor. The household cuts back on n_t^s to reduce the probability of susceptible people becoming infected. Consistent with this logic, the reduction in n_t^s mirrors the path for I_t.

The household has an incentive to smooth consumption over time because consuming does not increase anyone's probability of becoming infected. Infected and recovered people are not affected by exposure to the virus. So, to smooth consumption over time and across people, the household increases n_t^i and n_t^r.

The income of susceptible people falls dramatically. But their consumption does not, so their savings turn sharply negative. The household finances that dissaving by a massive decline in investment. In effect, investment allows the household to smooth consumption and hours worked in response to a transitory fall in n_t^s.

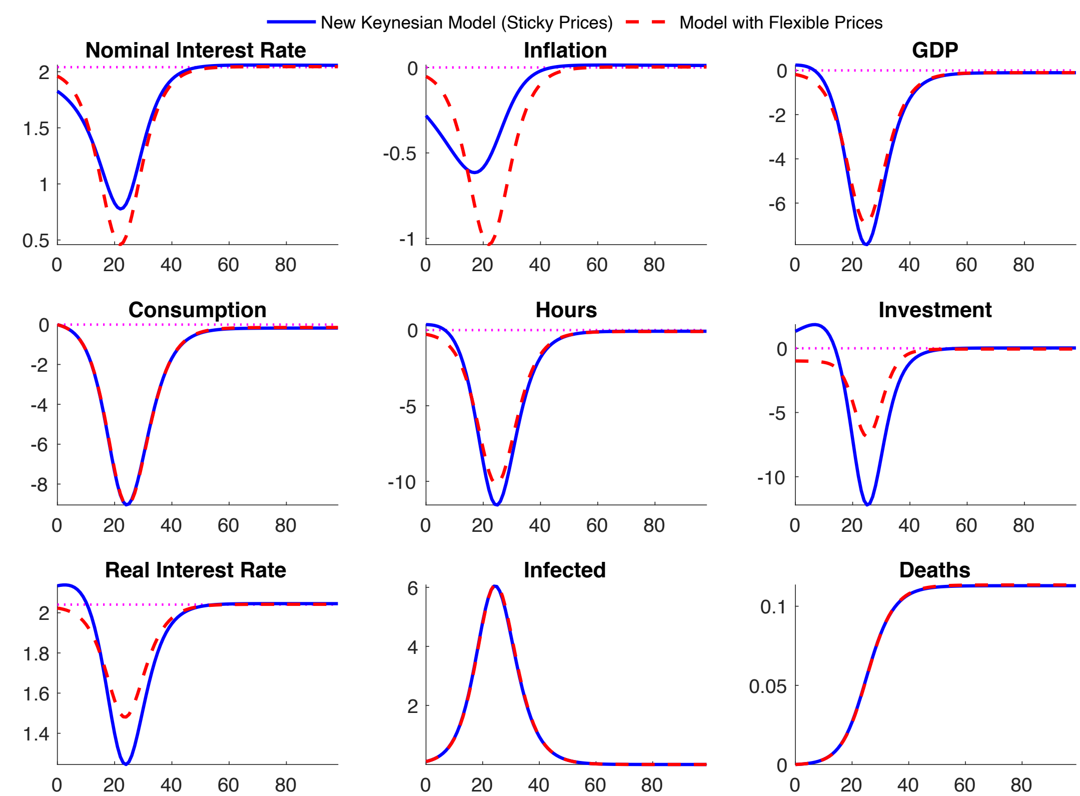
In sum, when $\pi_1 = 0$, the epidemic causes a large recession. But, with this parameterization, the model cannot rationalize a key feature of the COVID-19 recession: the large observed decline in consumption.

4.3 Epidemics as a shock to the demand for consumption *and* the supply of labor

In our benchmark calibration, both π_1 and π_2 are positive. So an epidemic acts like a negative shock to both consumption demand and labor supply.[14]

Figure 9 displays the total impact of the epidemic on key macro variables. This figure shows that the model captures the salient features of the epidemic recession. There is a large drop in output, consumption, investment, and hours worked with peak to trough declines of 8, $9, 12$, and 12 percent, respectively. The drop in consumption reflects the fall in consumption demand by susceptible people. The large fall in investment reflects the importance of the labor supply shock. As in the data, the epidemic recession is associated with mild deflation.

Figure 9: Epidemic as a shock to consumption demand and labor supply



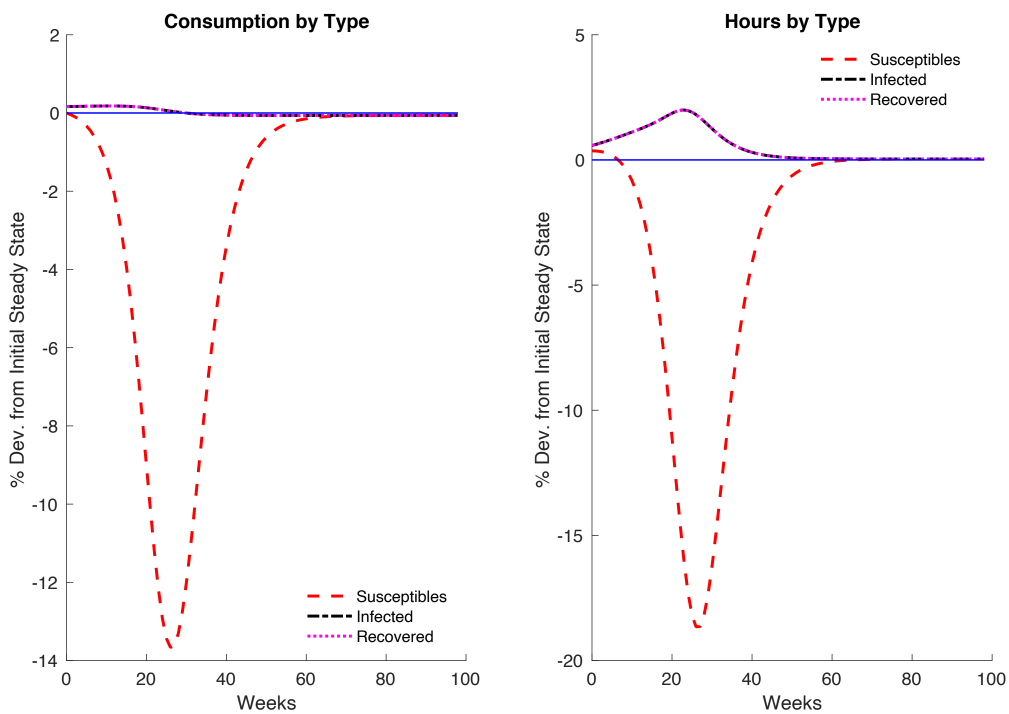
Notes: x-axis in weeks. GDP, consumption, hours and investment in percent deviations from initial steady state. Inflation, nominal and real interest rates in percent. Infected and deaths in percent of initial population.

Figure 10 shows the individual responses of consumption and hours worked of people

[14] While this decomposition is useful for intuition, the quantitative impact is not the simple sum of the two shocks given the nonlinear nature of the model.

with different health statuses. We see that susceptible people dramatically reduce their consumption and hours worked to reduce the probability of becoming infected. The labor income of susceptible people drops by more than their consumption. The resulting negative savings are feasible because infected and recovered people work more and consume roughly the same amount as they did before the epidemic.

Figure 10: Response of consumption and hours when epidemic is a shock to consumption demand and labor supply



To understand the last result, recall that the health status of infected and recovered people is not affected by exposure to the virus. So, the household wants to keep their consumption relatively smooth while increasing their labor supply as part of the risk-sharing arrangement within the family.

Finally, Figure 9 also displays the impact of an epidemic in a version of the model in which prices are flexible. We see that the recession is slightly larger when prices are sticky. The key difference between the two models concerns inflation. The flexible price model predicts a larger fall in prices than the sticky price model.

5 Conclusion

We analyze the effects of an epidemic in an NK model. We show that this model can rationalize the positive comovement of consumption, investment, and output and the moderate deflation observed in the recessions associated with the Covid epidemic in six developed countries. A natural next step is to embed an epidemiological model into a DSGE model to evaluate the various policy interventions implemented during the Covid recession.

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Appendix A Equilibrium equations

We have the following 31 endogenous variables:

$$y_t, k_t, n_t, w_t, r_t^k, x_t, c_t, s_t, i_t, r_t, n_t^s, n_t^i, n_t^r,$$
$$c_t^s, c_t^i, c_t^r, \tau_t, \tilde{\lambda}_t^b, \lambda_t^\tau, \lambda_t^i, \lambda_t^s, \lambda_t^r, d_t, pop_t$$
$$\breve{p}_t, mc_t, rr_t, R_t^b, \pi_t, K_t^f, F_t,$$

and the following 31 equilibrium conditions

$$
\begin{aligned}
1)\ y_t &= \breve{p}_t A k_t^{1-\alpha} n_t^\alpha \\
2)\ mc_t &= \frac{w_t^\alpha \left(R_t^k / P_t\right)^{1-\alpha}}{A\alpha^\alpha (1-\alpha)^{1-\alpha}} \\
3)\ w_t &= \frac{W_t}{P_t} = mc_t \alpha A n_t^{\alpha-1} k_t^{1-\alpha} \\
4)\ k_{t+1} &= x_t + (1-\delta) k_t \\
5)\ y_t &= c_t + x_t + g \\
6)\ n_t &= s_t n_t^s + i_t n_t^i + r_t n_t^r \\
7)\ c_t &= s_t c_t^s + i_t c_t^i + r_t c_t^r \\
8)\ \tau_t &= \pi_1 s_t c_t^s \left(i_t c_t^i\right) + \pi_2 s_t n_t^s \left(i_t n_t^i\right) + \pi_3 s_t i_t \\
9)\ s_{t+1} &= s_t - \tau_t \\
10)\ i_{t+1} &= i_t + \tau_t - (\pi_r + \pi_d)\, i_t \\
11)\ r_{t+1} &= r_t + \pi_r i_t \\
12)\ d_{t+1} &= d_t + \pi_d i_t, \\
13)\ pop_{t+1} &= pop_t - \pi_d i_t, \\
14)\ \frac{1}{c_t^s} &= \tilde{\lambda}_t^b - \lambda_t^\tau \pi_1 \left(i_t c_t^i\right) \\
15)\ \frac{1}{c_t^i} &= \tilde{\lambda}_t^b \\
16)\ \frac{1}{c_t^r} &= \tilde{\lambda}_t^b \\
17)\ \theta n_t^s &= \tilde{\lambda}_t^b w_t + \lambda_t^\tau \pi_2 \left(i_t n_t^i\right) \\
18)\ \theta n_t^i &= \tilde{\lambda}_t^b w_t \\
19)\ \theta n_t^r &= \tilde{\lambda}_t^b w_t
\end{aligned}
$$

$$
\begin{aligned}
20)\ \tilde{\lambda}_t^b &= \beta(r_{t+1}^k + 1 - \delta)\tilde{\lambda}_{t+1}^b \\
21)\ \lambda_t^i &= \lambda_t^\tau + \lambda_t^s \\
22)\ 0 &= \log(c_{t+1}^s) - \frac{\theta}{2}\left(n_{t+1}^s\right)^2 + \lambda_{t+1}^\tau \left[\begin{array}{c} \pi_1 c_{t+1}^s \left(i_{t+1} c_{t+1}^i\right) \\ +\pi_2 n_{t+1}^s \left(i_{t+1} n_{t+1}^i\right) + \pi_3 i_{t+1} \end{array} \right] \\
&\quad + \tilde{\lambda}_{t+1}^b \left[w_{t+1} n_{t+1}^s - c_{t+1}^s\right] - \lambda_t^s/\beta + \lambda_{t+1}^s \\
23)\ 0 &= \log(c_{t+1}^i) - \frac{\theta}{2}\left(n_{t+1}^i\right)^2 + \tilde{\lambda}_{t+1}^b \left[w_{t+1} n_{t+1}^i - c_{t+1}^i\right] \\
&\quad - \lambda_t^i/\beta + \lambda_{t+1}^i (1 - \pi_r - \pi_d) + \lambda_{t+1}^r \pi_r \\
24)\ 0 &= \log(c_{t+1}^r) - \frac{\theta}{2}\left(n_{t+1}^r\right)^2 + \tilde{\lambda}_{t+1}^b \left[w_{t+1} n_{t+1}^r - c_{t+1}^r\right] - \lambda_t^r/\beta + \lambda_{t+1}^r \\
25)\ \tilde{\lambda}_t^b &= \beta r r_t \tilde{\lambda}_{t+1}^b \\
26)\ r r_t &= \frac{R_t^b}{\pi_{t+1}}.
\end{aligned}
$$

The first-order conditions for optimal price setting are:

$$
\begin{aligned}
27)\ K_t^f &= \gamma m c_t \tilde{\lambda}_t^b y_t + \beta \xi \pi_{t+1}^{\frac{\gamma}{\gamma-1}} K_{t+1}^f \\
28)\ F_t &= \tilde{\lambda}_t^b y_t + \beta \xi \pi_{t+1}^{\frac{1}{\gamma-1}} F_{t+1} \\
29)\ K_t^f &= F_t \left(\frac{1 - \xi \pi_t^{\frac{1}{\gamma-1}}}{1 - \xi} \right)^{-(\gamma-1)}.
\end{aligned}
$$

The inverse price dispersion term is given by:

$$
30)\ \breve{p}_t = \left[(1-\xi) \left(\frac{1 - \xi \pi_t^{\frac{1}{\gamma-1}}}{1-\xi} \right)^{\gamma} + \xi \frac{\pi_t^{\frac{\gamma}{\gamma-1}}}{\breve{p}_{t-1}} \right]^{-1}.
$$

Finally, the Taylor rule is given by:

$$
31)\ R_t^b - R^b = r_\pi \log \frac{\pi_t}{\pi} + r_x \log(y_t / y_t^f).
$$

Here, y_t^f is flexible price output which can be computed using equations $1) - 31)$ setting $\xi = 0$.

In equations $1) - 31)$ $\tilde{\lambda}_t^b$ is the scaled Lagrange multiplier, i.e. $\tilde{\lambda}_t^b = \lambda_t^b P_t$.

We solve the nonlinear equilibrium equations $1) - 31)$ as well as their flexible price version using a gradient-based two-point boundary-value algorithm.